Cycle-Based Singleton Local Consistencies

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Motivation

High-level consistency effectively prune search space but can be costly

Contributions

1. Exploit cycles in the constraint network of a Constraint Satisfaction Problem (CSP) to vehicle constraint propagation
2. Focus: Enforce POAC on a Minimum Cycle Basis (MCB) of the incidence graph of the CSP
3. Empirically show benefit

1. Local Consistency

Variables: $\{x_1, x_2, x_3, x_4, x_5, x_6\}$
Domains: $\{v_0, v_1, v_2, v_3\}$ for x_1, x_2, x_3; $\{v_1, v_2\}$ for x_4, x_5, x_6
Constraints:

R_{12}	
x_1	x_2
v_0	v_0
v_1	v_1
v_2	v_2
v_3	v_1
v_3	v_2

R_{13}	
x_1	x_3
v_0	v_0
v_1	v_1
v_1	v_3
v_2	v_2
v_2	v_3
v_3	v_1
v_3	v_2
v_3	v_3

R_{14}	
x_1	x_4
v_0	v_1
v_1	v_1
v_1	v_2
v_2	v_1
v_2	v_2
v_3	v_2

R_{23}	
x_2	x_3
v_0	v_1
v_1	v_0
v_1	v_2
v_1	v_3
v_2	v_1
v_2	v_3

R_{34}	
x_3	x_4
v_0	v_1
v_1	v_1
v_1	v_2
v_2	v_1
v_2	v_2
v_3	v_2

R_{156}		
x_1	x_5	x_6
v_0	v_1	v_1
v_1	v_1	v_1
v_2	v_2	v_2
v_3	v_3	v_3

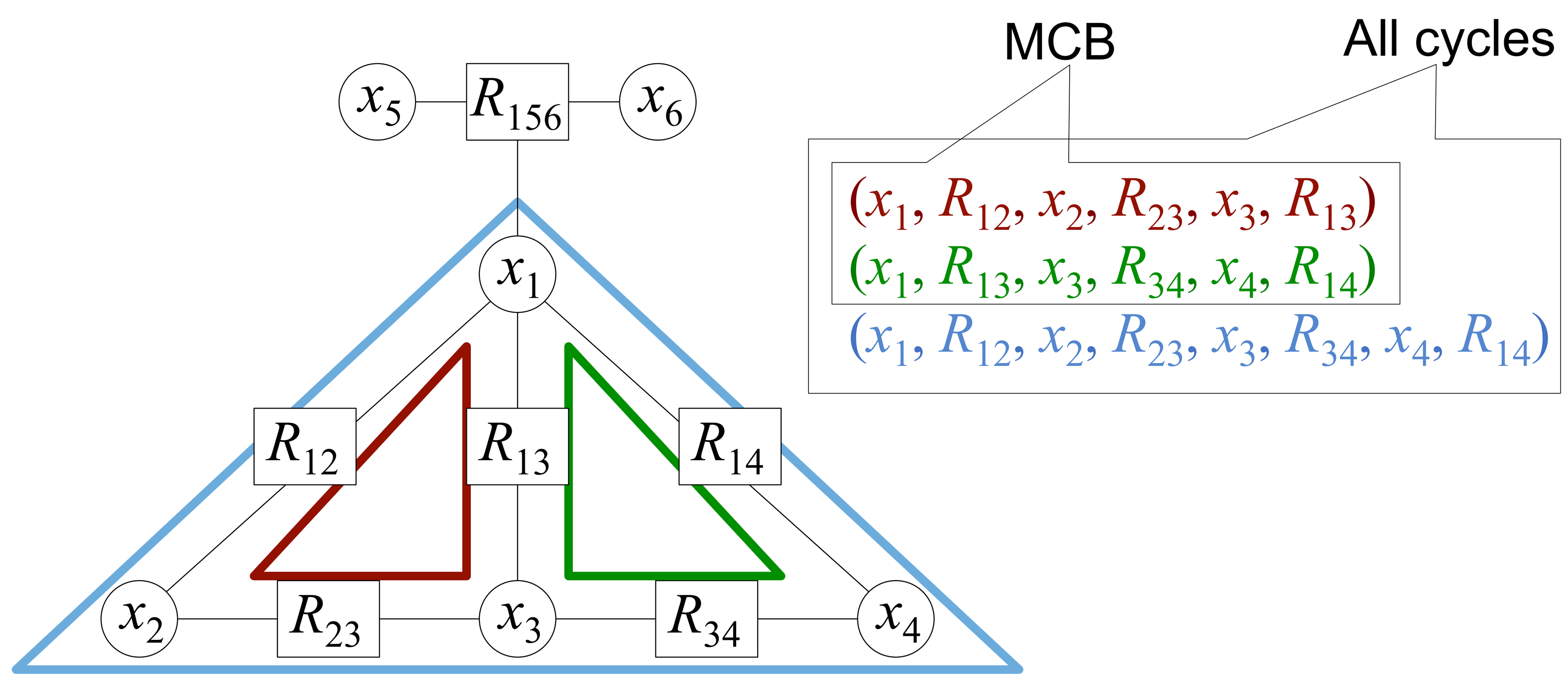
Generalized Arc Consistency (GAC) ensures a value in the domain of a variable in the scope of a relation can be extended to a tuple satisfying the relation.
E.g., v_3 can be removed from x_2

Singleton Arc Consistency (SAC) ensures that the CSP remains arc consistent after assigning a value to a variable.
E.g., v_0 can be removed from x_1, x_2, x_3

A constraint network $\mathcal{P} = (\mathcal{X}, \mathcal{D}, \mathcal{C})$ is **Partition-One Arc-Consistent** (POAC) iff $\mathcal{P}$ is SAC and for all $x_i \in \mathcal{X}$, for all $v_i \in \mathrm{dom}(x_i)$, for all $x_j \in \mathcal{X}$, there exists $v_j \in \mathrm{dom}(x_j)$ such that $(x_i, v_i) \in \mathrm{AC}(\mathcal{P} \cup \{x_j \leftarrow v_j\})$ [Bennaceur and Affane CP 2001]
E.g., v_1 can be removed from x_4 because there is no such v_j for x_1 where $(x_4, v_1) \in \mathrm{AC}(\mathcal{P} \cup \{x_1 \leftarrow v_j\})$.

2. Minimum Cycle Basis

Computed on incidence graph, bipartite graph $G = (\mathcal{X}, \mathcal{C}, E)$
$\mathcal{X}$: variables, $\mathcal{C}$: constraints, E: link c_i and $x_i \in scope(c_i)$

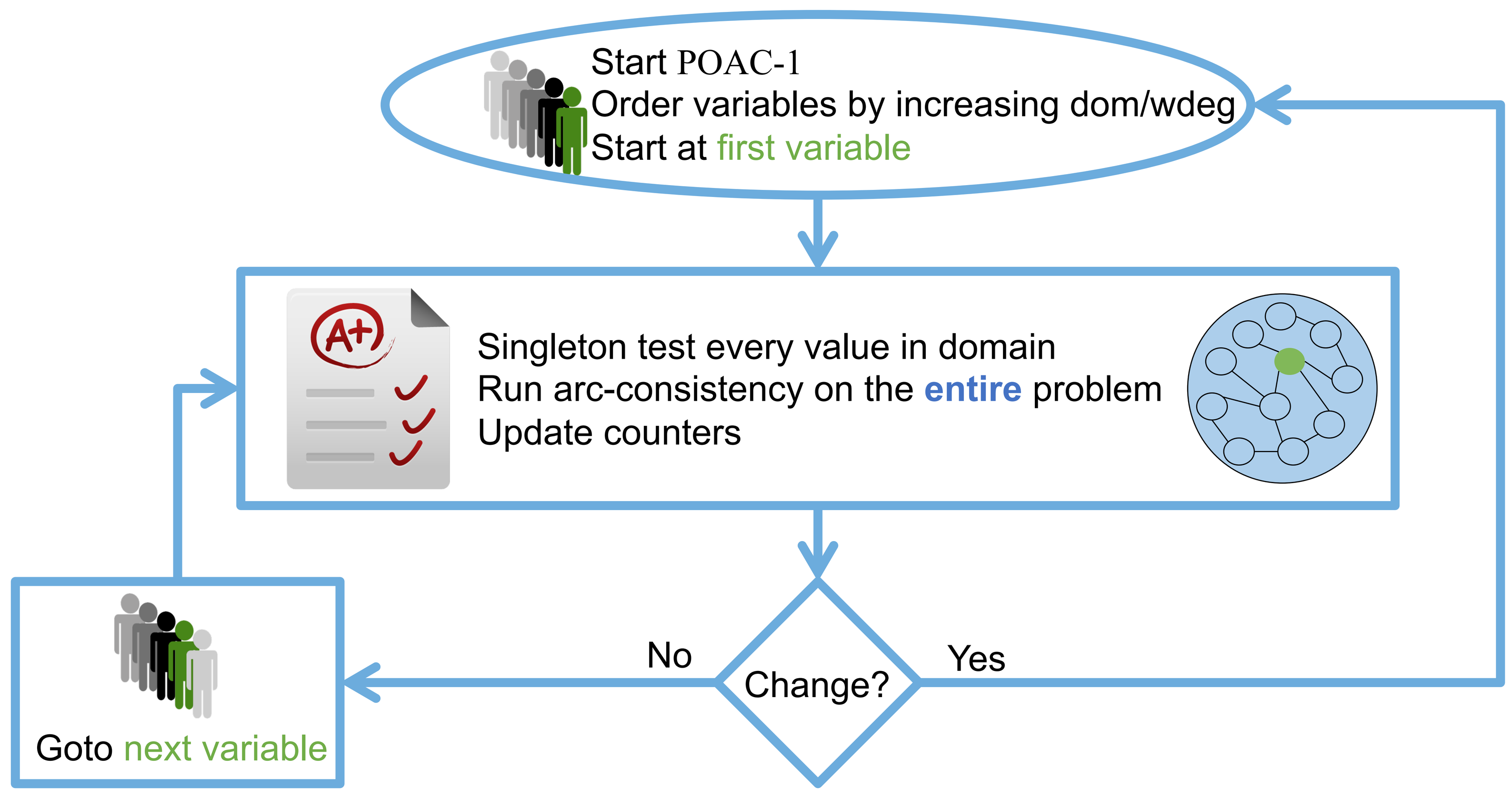


A **Minimum Cycle Basis** (MCB): a cycle ∈ MCB cannot be obtained by symmetric difference from other cycles ∈ MCB

3. Localizing POAC

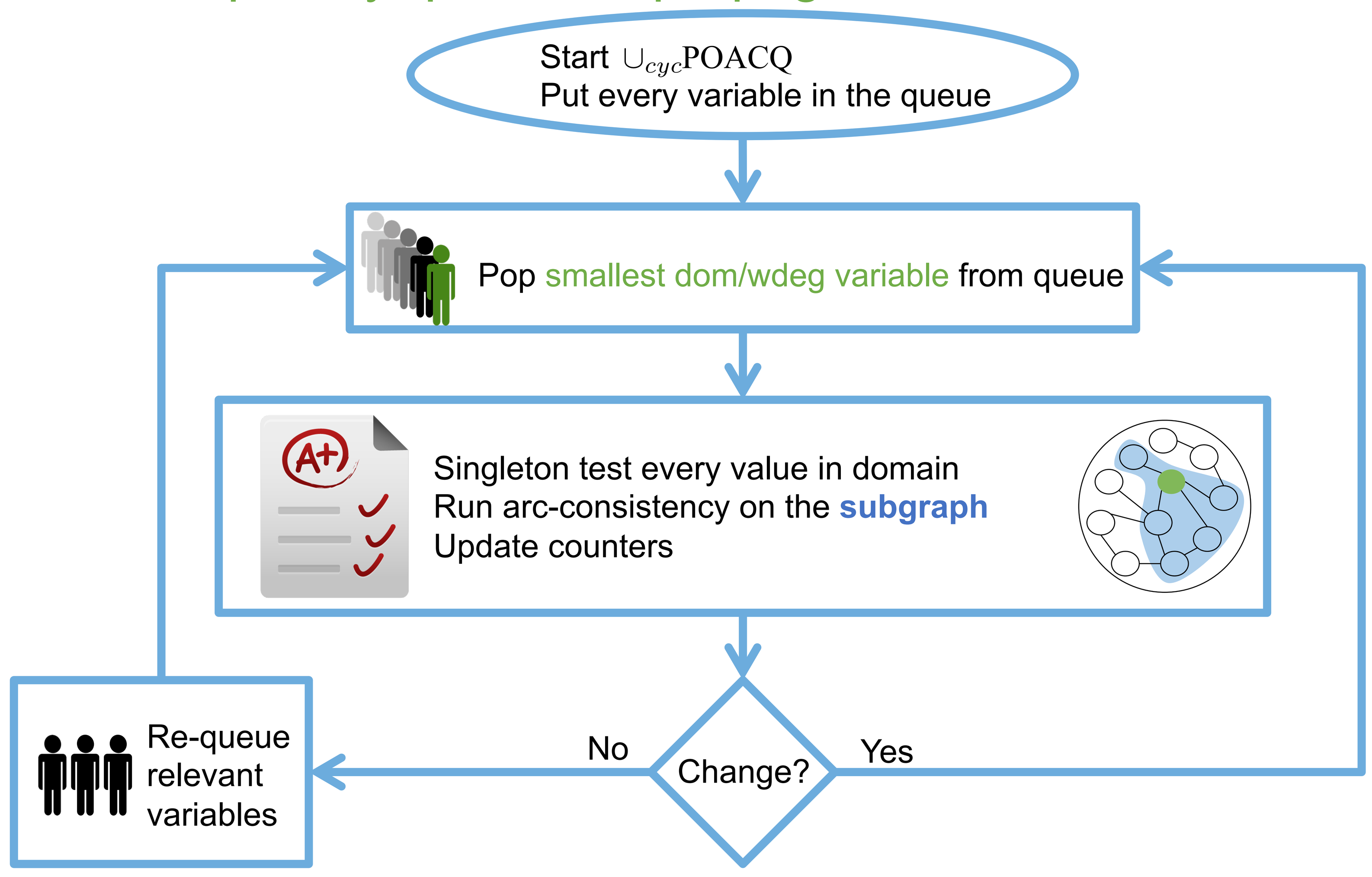
Union-Cycle POAC ($\cup_{cyc}$POAC): restrict the singleton test to the neighborhood of a variable and to the union of the MCB cycles in which the variable appears.

Algorithm derived from POAC-1 [Balafrej+ AAAI 2014]



We introduce $\cup_{cyc}$POACQ, derived from POAC-1, with two major changes:

1. Restrict the singleton test to the neighborhood union of the MCB cycles in which it appears (subgraph)
2. Use a priority queue for propagation



4. Empirical Evaluations

Benchmark		GAC	POAC	$\cup_{cyc}$POACQ	APOAC	A$\cup_{cyc}$POACQ
Adaptive POAC the best						
TSP-25	# solv	**15**	14	**15**	**15**	**15**
(# inst 15)	ΣCPU (s)	4,303.12	>41,382.27	32,654.67	6,152.91	**2,418.41**
cril	# solv	6	7	7	**8**	**8**
(# inst 8)	ΣCPU (s)	>30,458.10	>16,282.45	>16,651.04	2,321.96	**1,831.60**
QWH-20	# solv	**10**	**10**	**10**	**10**	**10**
(# inst 10)	ΣCPU (s)	2,256.61	6,154.43	3,007.98	2,236.32	**2,061.63**
k-insertions	# solv	17	17	**18**	**18**	**18**
(#inst 32)	ΣCPU (s)	>17,034.30	>21,639.31	11,814.83	**6,129.92**	8,940.59
Non-adaptive POAC the best						
mug	# solv	6	6	**8**	6	6
(# inst 8)	ΣCPU (s)	>54,724.38	>29,385.02	**13,655.87**	>34,207.98	>41,583.97
GAC the best						
TSP-20	# solv	**15**	**15**	**15**	**15**	**15**
(# inst 15)	ΣCPU (s)	**302.21**	2,750.90	3,096.07	593.04	384.13
renault	# solv	**50**	**50**	**50**	**50**	**50**
(# inst 50)	ΣCPU (s)	**55.87**	277.74	176.28	196.04	155.88
myciel	# solv	**13**	12	12	**13**	**13**
(# inst 16)	ΣCPU (s)	**1,711.93**	>21,564.06	>26,196.15	3,118.86	2,555.54

5. Future Research

Extend approach to other high-level consistency algorithms

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